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Z_s-Contractive Mappings and Weak Compatibility in Fuzzy Metric Space

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Abstract. The aim of this paper is to introduce Z_s-contractive condition for a pair of self maps in a fuzzy metric space, which enlarges and unifies the existing fuzzy contractions (by Gregori and Sapena [4]), ψ-contraction (by Mihet [8]), Z-contractions (by Shukla [15]) and Tirado contraction ([16]), which are for only one self map. Using this, we establish a unique common fixed point theorem for two self maps satisfying condition (S), which was introduced by Shukla et al. in [15] through weak compatibility. The article includes an example, which shows the validity of our results.

1. Introduction

In 1965 L. Zadeh [18] itroduce the theory of fuzzy sets. Later on in 1978 the concept of fuzzy metric space was introduced by Kramosil and Michalek in [6], which was modified by George and Veeramani [2] in order to obtain a Hausdorff topology for this class of fuzzy metric spaces. Contractive mappings in fuzzy metric spaces were studied by various authors (see, e.g., Gregori and Sapena [4], Mihet [8], Tirado [16] and Wardowski [17] and Shukla et al [15]) and used in establishing some fixed point theorems in fuzzy metric space in the sense of George and Veeramani.

Recently, in [15] Shukla et al. introduced a new class of contractive mappings called Z-contractions, and proved some fixed point results for a self map of this new class. Motivated by that paper we introduce Z_s-contraction for two self maps in the setting of fuzzy metric space, which enlarges and unifies the existing contraction in the right sense. Employing condition (S), we prove the existence of unique common fixed point of a pair of Z_s-contractive self maps in fuzzy metric space through weak compatibility.

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The structure of the paper is as follows:

After preliminaries in section 3, we introduce Z_s-contraction. Then we study fuzzy contractive mapping due to Gregori et al. [4], Mihet [8], Tirado [16] and Wardowski [17]. In section 4, we prove the existence of unique common fixed point of a Z_s-contractive pair of self maps satisfying condition (S).

2. Preliminaries

Definition 2.1 ([12]). A mapping $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous triangular norm (t-norm for short) if $*$ is continuous and satisfies the following conditions:

(i) $*$ is commutative and associative, i.e., $a * b = b * a$ and $a * (b * c) = (a * b) * c$, for all $a, b, c \in [0,1]$;
(ii) $1 * a = a$, for all $a \in [0,1]$;
(iii) $a * c \leq b * d$, for $a \leq b, c \leq d$ for $a, b, c, d \in [0,1]$.

Well known examples of t-norm are, the minimum t-norm $*, a * b = \min\{a, b\}$ and product t-norm $*, a * b = ab$.

Definition 2.2 ([2]). A fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a (nonempty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $t, s > 0$;

(GV1) $M(x, y, t) > 0$;
(GV2) $M(x, y, t) = 1$ if and only if $x = y$;
(GV3) $M(x, y, t) = M(y, x, t)$;
(GV4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$;
(GV5) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Note that in view of condition (GV2) we have $M(x, x, t) = 1$, for all $x \in X$ and $t > 0$ and $M(x, y, t) < 1$, for all $x \neq y$ and $t > 0$.

The following notion was introduced by George and Veeramani in [2] (and previously, by H. Sherwood, in the context of PM-spaces [13]).

Definition 2.3 ([2, 12]). A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be M-Cauchy, or simply Cauchy, if for each $\epsilon \in (0,1)$ and each $t > 0$ there exists an $n_0 \in N$, such that $M(x_n, x_m, t) > 1 - \epsilon$, for all $n, m \geq n_0$. Equivalently, $\{x_n\}$ is Cauchy if $\lim\limits_{\substack{n \to \infty \\ m \to \infty}} M(x_n, x_m, t) = 1$, for all $t > 0$.

Remark 2.1. If $\lim\limits_{n \to \infty} \inf\limits_{m > n} M(x_n, x_m, t) = 1$, for all $t > 0$, then $\{x_n\}$ is M-Cauchy.

Theorem 2.1 ([2]). *Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}_{n \in N}$ in X converges to $x \in X$ if and only if $\lim\limits_{n \to \infty} M(x_n, x, t) \to 1$.*

Definition 2.4 ([2])**.** $(X, M, *)$ (or simply X) is called M-complete if every Cauchy sequence in X is convergent.

Definition 2.5 ([14])**.** Let S and T mappings on a fuzzy metric space $(X, M, *)$ into itself. The mappings are said to be weak compatible if they commute at their coincidence points, i.e., $Sx = Tx$ implies $STx = TSx$.

3. Z_s-CONTRACTION

Let Z_s denotes the family of functions $\zeta : (0, 1) \times (0, 1] \to \Re^+$, where $\Re^+ = (0, \infty)$ satisfying the following condition:

$$\begin{aligned} \zeta(t, s) &> s, \text{ if } t, s \in (0, 1), \\ &= 1, \text{ if } t \in (0, 1), s = 1. \end{aligned}$$

Example 3.1. Define $\zeta : (0, 1) \times (0, 1] \to \Re^+$ by

$$\zeta(t, s) = \begin{cases} \frac{s}{t} & \text{if } t \geq s; \\ \frac{t}{s}, & \text{if } s \geq t; \\ 1, & \text{if } s = 1, t \in (0, 1). \end{cases}$$

Example 3.2. Define $\zeta : (0, 1) \times (0, 1] \to \Re^+$ by

$$\zeta(t, s) = \begin{cases} \frac{1}{s+t} + t; & \text{if } s, t \in (0, 1) \\ 1, & \text{if } s = 1, t \in (0, 1). \end{cases}$$

Definition 3.1. Let $(X, M, *)$ be a fuzzy metric space. A pair (A, B) of self maps in X is said to be fuzzy Z_s-contractive if there exists $\zeta \in Z$ such that for all $x, y \in X$ with $Ax \neq Ay$ and for all $t > 0$,

$$M(Ax, Ay, t) \geq \zeta(M(Ax, Ay, t), M(Bx, By, t)). \tag{1}$$

Remark 3.1. If $x, y \in X$ and $Ax \neq Ay$, and $B = I$ then

$$M(Ax, Ay, t) \geq \zeta(M(Ax, Ay, t), M(x, y, t)), \text{ for all } t > 0, \tag{2}$$

which is precisely the Z-contraction, for a self map given by Shukla et al. [15].

Gregori and Sapena in [4] defined the fuzzy contractive mappings as follows:

Let $(X, M, *)$ be a fuzzy metric space. A mapping $T : X \to X$ is called a fuzzy contractive mapping if there exists $k \in (0, 1)$ such that

$$\frac{1}{M(Tx, Ty, t)} - 1 \leq k \left(\frac{1}{M(x, y, t)} - 1 \right),$$

for all $x, y \in X$.

Remark 3.2. The fuzzy contractive mapping defined by Gregori and Sapena in [4] is Z_s-contractive if we take $\zeta \in Z_s$ to be

$$\zeta(t, s) = \frac{s}{k + (1 - k)s}, \text{ for all } t \in (0, 1), \text{ for all } s \in (0, 1] \text{ in (2)}.$$

We note that $\zeta(t,1) = 1$, here.

In [16], Tirado's defined the following contraction:

Let $(X, M, *)$ be a fuzzy metric space. A mapping $T : X \to X$ is Tirado contraction if there exists $k \in (0,1)$ such that

$$1 - M(Tx, Ty, t) \leq k(1 - M(x, y, t)), \text{ for all } x, y \in X.$$

Remark 3.3. Every Tirado contraction is a is Z_s-contractive if we take $\zeta \in Z_s$ to be

$$\zeta(t, s) = 1 + k(s - 1), \text{ for all } t \in (0, 1), \text{ for all } s \in (0, 1] \text{ in (2)}.$$

We note that $\zeta(t,1) = 1$, here.

In [8], Mihet, defined the a class Ψ of mappings as follows:

Let $\psi : (0, 1] \to (0, 1]$ such that ψ is continuous, nondecreasing and $\psi(t) > t, \forall t \in (0, 1)$. Let $\psi \in \Psi$. A mapping $T : X \to X$ is called a fuzzy ψ-contractive mapping if:

$$M(x, y, t) > 0 \Rightarrow M(Tx, Ty, t) \geq \psi(M(x, y, t)),$$

for all $x, y \in X$ and $t > 0$.

Remark 3.4. Every fuzzy ψ-contractive mapping is Z_s-contractive, if we take $\zeta \in Z_s$ to be

$$\zeta(t, s) = \psi(s), \text{ for all } t \in (0, 1), \text{ for all } s \in (0, 1] \text{ in (2)}.$$

We note that $\zeta(t,1) = 1$, here.

In [17], Wardowski defined the following class H of mappings as follows:

Let H be the family of the mappings $\eta : (0, 1] \to [0, \infty)$ satisfying the following conditions:

(H-1) η transforms $(0, 1]$ onto $[0, \infty)$;
(H-2) η is strictly decreasing.

A mapping $T : X \to X$ is called fuzzy H-contractive with respect to $\eta \in H$ if there exists $k \in (0, 1)$ satisfying the following condition:

$$\eta(M(Tx, Ty, t)) \leq k\eta(M(x, y, t),$$

for all $x, y \in X$ and $t > 0$.

Remark 3.5. Inview of the remark by Gregori and Minana in [5], every fuzzy H-contractive mapping with respect to $\eta \in H$ is a fuzzy Z_s-contractive with respect to the function $\zeta \in Z_s$ if we define $\zeta(t, s) = \eta^{-1}(k\eta(s))$, for all $t \in (0, 1), s \in (0, 1]$ in equation (2). We note that $\zeta(t, 1) = 1$ here as $\eta(1) = 0$.

Definition 3.2. Let $(X, M, *)$ be a fuzzy metric space and A and B be self-maps in X which are Z_s-contractive with respect to $\zeta \in Z_s$. The Quintuplet

(X, M, A, B, ζ) said to satisfy property (S), if for a sequence $\{y_n\}$, with initial point $x_0, Ax_n = Bx_{n+1} = y_n$, for $n = 0, 1, 2, \ldots$

$$\inf_{m>n} M(y_n, y_m, t) \leq \inf_{m>n} M(y_{n+1}, y_{m+1}, t), \text{ for all } n > m,$$

implies

$$\lim_{n\to\infty} \inf_{m>n} \zeta\big(M(y_{n+1}, y_{m+1}, t), M(y_n, y_m, t)\big) = 1.$$

4. Main Results

Our first new result is the next:

Theorem 4.1. *Let A and B be self maps in a fuzzy metric space $(X, M, *)$ satisfying the following conditions:*

(4.11) $A(X) \subseteq B(X)$
(4.12) *The pair (A, B) is Z_s-contractive.*
(4.13) *$B(X)$ is complete.*
(4.14) *The pair (A, B) is weakly compatible.*
(4.15) *The Quintuplet (X, M, A, B, ζ) satisfies property (S).*

Then A and B have a unique common fixed point in X.

Proof. Let $x_0 \in X$, be any arbitrary point. Construct sequence $\{y_n\}, Ax_n = Bx_{n+1} = y_n$, for $n = 0, 1, 2, \ldots$. First we show that if the two maps A and B have a common fixed point then it is unique. Let u and v be two common fixed points of A and B. Then $u = Au = Bu$ and $v = Av = Bv$.

We show that $u = v$. Suppose, on the contrary that $u \neq v$, then $Au \neq Av$. Now,

$$\begin{aligned} M(u, v, t) &= M(Au, Av, t), \\ &\geq \zeta(M(Au, Av, t), M(Bu, Bv, t)), \text{ (using (1))} \\ &> M(u, v, t), \end{aligned}$$

i.e., $M(u, v, t) > M(u, v, t)$, which is a contradiction. So $u = v$. Thus, if the pair (A, B) has a common fixed point then it is unique.

Now we prove the existence of common fixed point of self maps A and B.

CASE I: Suppose $y_n = y_{n+1}$, for some $n \in N$. Now $y_n = Ax_n = Bx_{n+1} = Ax_{n+1} = Bx_{n+2} = y_{n+1}$ we have $Ax_{n+1} = Bx_{n+1}$. Let $Ax_{n+1} = Bx_{n+1} = z$. So x_{n+1} is a point of coincidence of the pair (A, B). As the pair (A, B) is weakly compatible we have $Az = Bz$. Now we show that $Az = z$. Suppose, if possible on the contrary, that $Az \neq z$.

$$\begin{aligned} M(z, Az, t) &= M(Ax_{n+1}, Az, t), \\ &\geq \zeta(M(Ax_{n+1}, Az, t), M(Bx_{n+1}, Bz, t)), \text{ (using (1))} \\ &> M(Bx_{n+1}, Bz, t), \\ &= M(z, Bz, t), \\ &= M(z, Az, t), \end{aligned}$$

i.e.,

$$M(z, Az, t) > M(z, Az, t),$$

for all $t > 0$, which is not possible. Hence $Az = z$. So z is a common fixed point of the pair (A, B) in this case.

So we can assume the consecutive terms of the sequence $\{y_n\}$ are distinct.

Again, to see the existence of common fixed point in other cases, we first show that all the terms of the the sequence $\{y_n\}$ are distinct.

CASE II: Suppose $y_n = y_m$, for some $m > n$, and no two consecutive terms of the sequence $\{y_n\}$ are equal. Then we claim that $y_{n+1} = y_{m+1}$. Suppose, if possible, on the contrary, that $y_{n+1} \neq y_{m+1}$. Now

$$\begin{aligned} M(y_{n+1}, y_{m+1}, t) &= M(Ax_{n+1}, Ax_{m+1}, t) \\ &\geq \zeta(M(Ax_{n+1}, Ax_{m+1}, t), M(Bx_{n+1}, Bx_{m+1}, t)) \\ &= \zeta(M(y_{n+1}, y_{m+1}, t), M(y_n, y_m, t)) \\ &= 1, \end{aligned}$$

which is not possible, as L.H.S. is less than 1, hence the claim.

Also,

$$\begin{aligned} M(y_{n+1}, y_{n+2}, t) &= M(Ax_{n+1}, Ax_{n+2}, t) \\ &\geq \zeta(M(Ax_{n+1}, Ax_{n+2}, t), M(Bx_{n+1}, Bx_{n+2}, t)) \\ &= \zeta(M(y_{n+1}, y_{n+2}, t), M(y_n, y_{n+1}, t)) \\ &> M(y_n, y_{n+1}, t). \end{aligned}$$

$$\begin{aligned} M(y_n, y_{n+1}, t) &< M(y_{n+1}, y_{n+2}, t) \\ < M(y_{n+2}, y_{n+3}, t) &< \cdots < M(y_m, y_{m+1}, t), \end{aligned}$$

i.e., $M(y_n, y_{n+1}, t) < M(y_n, y_{n+1}, t)$, which is not possible. So this case does not arise.

So, we conclude that $y_n \neq y_m$ for distinct $n, m \in N$. Thus the elements of the sequence $\{y_n\}$ are distinct. Now we show that the sequence $\{y_n\}$ is M-Cauchy. For this we define an strictly increasing sequence $\{a_n\}$ by

$$a_n(t) = \inf_{m>n} \{M(y_n, y_m, t)\}, \text{ for } t > 0.$$

Now we show that $\{a_n\}$, converges to 1 as follows:

For this first we show that for $n > m$ we have

$$M(y_n, y_m, t) < M(y_{n+1}, y_{m+1}, t),$$

for all $m > n$. As no two terms of the sequence $\{y_n\}$ are equal we have

$$\begin{aligned} M(y_{n+1}, y_{m+1}, t) &= M(Ax_{n+1}, Ax_{m+1}, t) \\ &\geq \zeta(M(Ax_{n+1}, Ax_{m+1}, t), M(Bx_{n+1}, Bx_{m+1}, t)) \text{ (using (1))} \\ &= \zeta(M(y_{n+1}, y_{m+1}, t), M(y_n, y_m, t)) \\ &> M(y_n, y_m, t), \end{aligned}$$

thus

$$M(y_n, y_m, t) < M(y_{n+1}, y_{m+1}, t), \text{ for all } m > n.$$

This implies

$$\inf_{m>n} M(y_n, y_m, t) \leq \inf_{m>n} M(y_{n+1}, y_{m+1}, t), \tag{3}$$

i.e., $a_n(t) \leq a_{n+1}(t)$, for all $n \in N$. Thus, $\{a_n(t)\}$, for each $t > 0$, is an strictly increasing sequence, which is bounded above by 1. Let

$$\lim_{n\to\infty} a_n(t) = a(t), \text{ for } t > 0.$$

Now we claim that $a(t) = 1$.

Suppose, if possible, on the contrary, that $a(s) < 1$, for some $s > 0$. As the Quintuplet (X, M, A, B, ζ) has property (S), in view of equation (3) we have,

$$\lim_{n\to\infty} \inf_{m>n} \zeta(M(y_n, y_m, s), M(y_{n+1}, y_{m+1}, s)) = 1. \tag{4}$$

Again from equation (3) we have,

$$\begin{aligned} \inf_{m>n} M(y_{n+1}, y_{m+1}, s) &\geq \inf_{m>n} \zeta(M(y_{n+1}, y_{m+1}, s), M(y_n, y_m, s)), \\ &\geq \inf_{m>n} M(y_n, y_m, s). \end{aligned}$$

Letting $n \to \infty$ and using property (S) of the pair (A, B) in above equation we have

$$\lim_{n\to\infty} \inf_{m>n} M(y_n, y_m, s) = a(s) = 1,$$

which contradicts our hypothesis. hence the claim.

Now we show that the sequence $\{y_n\}$ is convergent in X and its limit is a fixed point of the maps A and B.

We have

$$a_n(t) = \inf_{m>n} M(y_n, y_m, t), \text{ for } t > 0.$$

$$\lim_{n\to\infty} a_n(t) = \lim_{n\to\infty} \inf_{m>n} M(y_n, y_m, t) = 1, \text{ for } t > 0,$$

i.e.,

$$\lim_{n\to\infty} \inf_{m>n} M(y_n, y_m, t) = 1, \text{ for } t > 0.$$

So by Remark 2.1, $\{y_n\}$ is an M-Cauchy sequence in $B(X)$ which is M-complete. Therefore there exists $u \in B(X)$ such that

$$\{y_n\} \to u, \tag{5}$$

i.e.,

$$\{Ax_n\} \to u \quad \text{and} \quad \{Bx_{n+1}\} \to u. \tag{6}$$

As $u \in B(X)$ there exists $v \in X$ such that

$$u = Bv. \tag{7}$$

STEP 1: Now we show that $Bv = Av$. Suppose, on the contrary, that $Av \neq Bv(= u)$. Then exists a positive integer n_0 such that $Bv \neq Bx_n$, for all $n \geq n_0$.

Taking $x = x_n$ and $y = v$ in equation (1) we get

$$\begin{aligned} M(Ax_n, Av, t) &\geq \zeta(M(Ax_n, Av, t), M(Bx_n, Bv, t)) \\ &> M(Bx_n, Bv, t)), \text{ if } Bx_n \neq v \end{aligned}$$

implies $M(Ax_n, Av, t) > M(Bx_n, Bv, t)$.

Letting $n \to \infty$ and using (6) and (7) we get

$$M(u, Av, t) \geq M(u, u, t) = 1.$$

Hence, $M(u, Bv, t) = 1$, which is not possible if $Bv \neq u$. Hence, $Bv = u$, and we have $Av = Bv = u$. As the pair of self maps (A, B) is weakly compatible, we have $Au = Bu$.

STEP 2:Now we show that $Au = u$. Suppose, on the contrary that $Au \neq u$. Then $Bu \neq u$. Taking $x = v$ and $y = u$ in equation (1) we get

$$\begin{aligned} M(Av, Au, t) &\geq \zeta(M(Av, Au, t), M(Bv, Bu, t)) \\ &> M(Bv, Bu, t), \\ &= M(u, Au, t), \end{aligned}$$

i.e., $M(u, Au, t) > M(u, Au, t)$ which is not possible.

Thus, $Au = Bu = u$. □

Taking $A = T$ and $B = I$ in Theorem 4.1, then the sequence $\{x_n\} = \{x_0, Tx_0, \ldots, T^n x_0, \ldots\}$ becomes a Picard sequence for the self map T and $\zeta \in Z_s$ becomes Z-contractive.

Corollary 4.1. *Let T be a self map on a M-complete fuzzy metric space $(X, M, *)$ with the following conditions:*

(4.11) *The map T is Z_s-contractive(which becomes Z contractive).*

(4.12) *The Quadruple (X, M, T, ζ) has property (S).*

Then the map T has a unique fixed point in X.

Remark 4.1. The above corollary is Theorem 3.13 of Shukla et al. [15].

Example 4.1. (of Theorem 3.1) Let $X = R^+$, the set of positive (> 0) real numbers and define a fuzzy set M on $X \times X \times (0, \infty)$ by:

$$M(x, y, t) = \begin{cases} 1, & \text{if } x = y, \\ \min\{x, y\}, & \text{if } x \neq y, \end{cases} \qquad x, y \in X, \ \ t \in (0, \infty).$$

Taking $a * b = a.b$ then $(X, M, *)$ is a M-complete fuzzy metric space. Define

$$\zeta(t, s) = \begin{cases} t, & \text{if } t > s, \\ \sqrt{s}, & \text{if } t \leq s, \end{cases} \quad s \in (0, 1], \ \ t \in (0, 1).$$

Then, $\zeta \in Z_s$.

Define self maps A and B on X by

$$A(x) = 1 - \frac{(x-1)^2}{2x}, \quad B(y) = 1 - \frac{(1-y)^2}{y}.$$

Consider a strictly increasing sequence $\{x_n\}$ of real numbers formed by $Ax_{2n} = x_{2n+1}, Bx_{2n+1} = x_{2n+2}$, for $n = 1, 2, \ldots$, with $x_0 = \frac{1}{2}$. Then $0 < x_n \leq 1$, for all $n \in N$ and $\lim_{n\to\infty} x_n = 1$.

Then quintuple (X, M, A, B, ζ) has the property (S). Also the pair $(A, B) \in Z_s$ with respect to the contraction ζ.

Thus, all the conditions of Theorem 4.1 are satisfied and $x = 1$ is the unique common fixed point of maps A and B.

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